New Achievable Rate Regions for Multi-terminal Classical-Quantum Channels via Simultaneous Decoding of Coset Codes

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Classical Channel

A classical discrete memoryless channel is described through:

- a finite input alphabet set $\mathcal{X}$
- a finite output alphabet set $\mathcal{Y}$
- and a transition probability $W_{Y|X}(y|x)$

Classical Quantum (CQ) channel

A classical quantum channel is described through:

- a finite input alphabet set $\mathcal{X}$
- a Hilbert space $\mathcal{H}$, for example $\mathcal{H} = \mathbb{C}^2$
- a collection of quantum state/density operators $\{\rho_x \in \mathcal{D}(\mathcal{H}) : x \in \mathcal{X}\}$

$$\mathrm{X} \longrightarrow \rho_x$$

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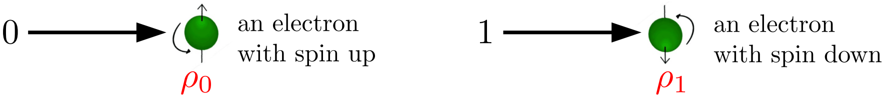
$$\mathrm{X} \longrightarrow \rho_x$$

Example of quantum state :

- polarized photon
- an electron with specific spin orientation

Suppose $\mathcal{X} = \{0, 1\}$



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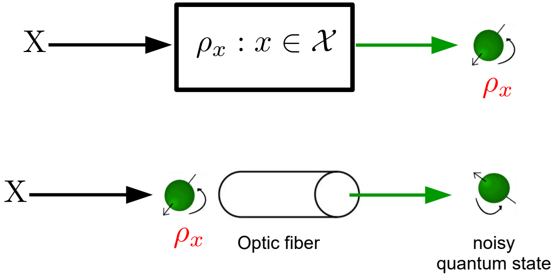

Key Difference

Classical

Bits are distinguishable
0 and 1 can be observed and distinguishable

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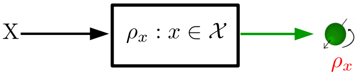
Quantum



Quantum states cannot be directly observed
ρ_0 and ρ_1 are indistinguishable

Key Difference

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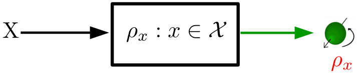
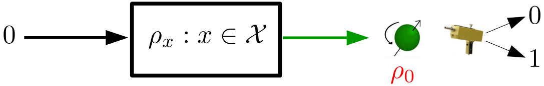
Quantum



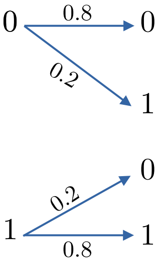
Quantum states are not directly observable objects.



The outcome of the measurement is random.

Key Difference

Classical

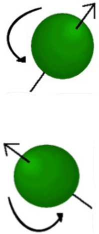


$W_{Y|X}(.|0)$

$W_{Y|X}(.|1)$

$W_{Y|X}(.|0)$ and $W_{Y|X}(.|1)$ are not distinguishable

Quantum



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Key Difference

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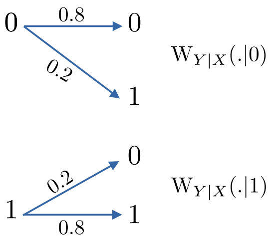
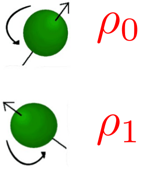


Quantum

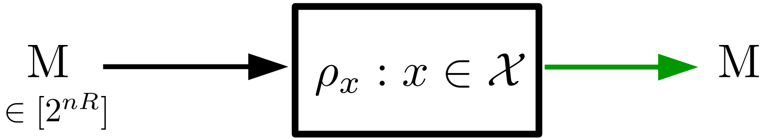


Build a codebook $(x^n(1), \ldots x^n(2^{nR}))$ such that $supp(W_{Y|X}(.|0))$ and $supp(W_{Y|X}(.|1))$ are disjoint.

Build a codebook $(x^n(1), \ldots x^n(2^{nR}))$ and a POVM $\{\lambda_m : m \in [2^{nR}]\}$ such that

$$\frac{1}{2^{nR}} \sum_m \operatorname{tr}\left\{(I - \lambda_m)\, \rho_{x^n(m)}\right\} \leq \epsilon.$$

Classical Quantum Point to Point Channel



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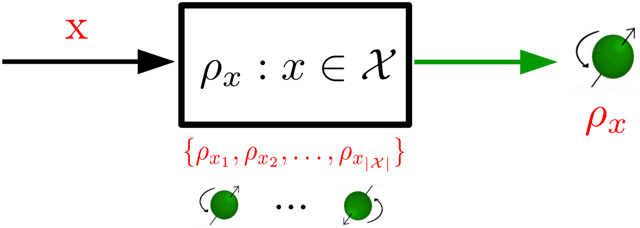

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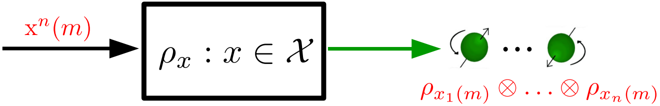

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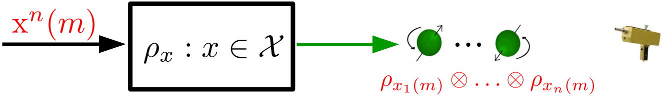

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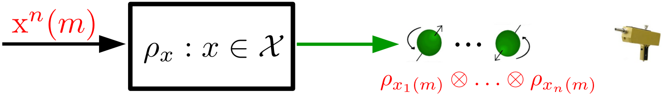
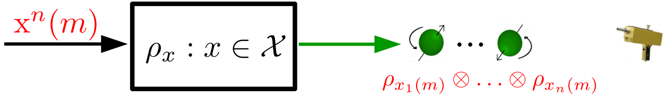


?? How many classical bits can we reliably communicate through the quantum channel ??

Classical Quantum Point to Point Channel



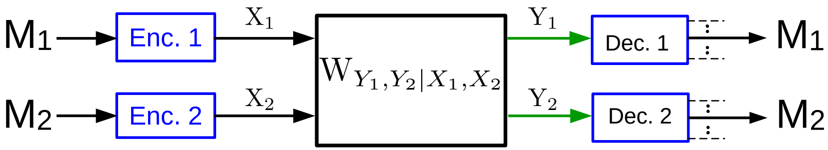
Design Codebook $= \{x^n(1), \cdots, x^n(2^{nR})\}$ and a POVM $\Lambda = \{\Lambda_1, \cdots, \Lambda_{2^{nR}}\}$ such that

$$\text{Avg. Prob. of error} = \frac{1}{2^{nR}} \sum_{m=1}^{2^{nR}} 1 - \mathrm{tr}\big(\Lambda_m \rho_{x^n(m)}\big) \leq \epsilon \text{ and } \frac{1}{n} \log(|\mathcal{M}|) \text{ is maximum}$$

2-User Classical Interference Channel

The 2-user classical interference channel is described through:

- two finite input alphabet sets : $\mathcal{X}_1, \mathcal{X}_2$
- two finite output alphabet sets : $\mathcal{Y}_1, \mathcal{Y}_2$
- and a transition probability : $W_{Y_1 Y_2 | X_1 X_2}(y_1, y_2 | x_1, x_2)$



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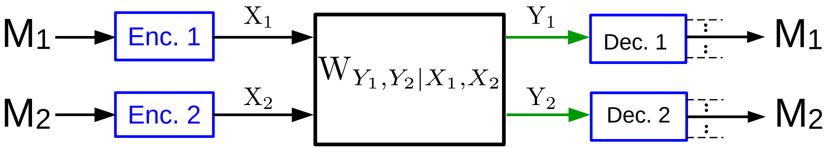
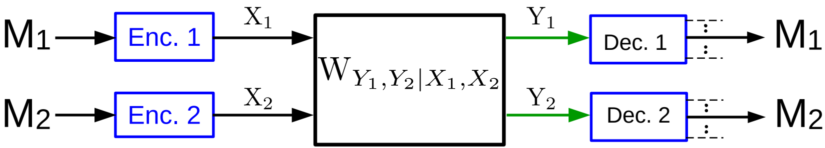


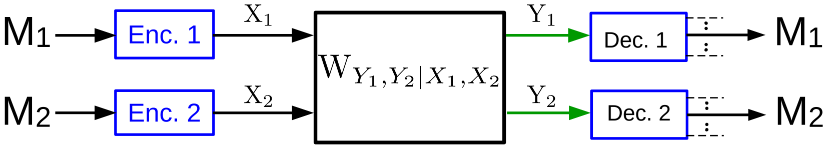
- Each Tx wants to send a message to its respective Rx.

2-User Classical Interference Channel



- Each Tx wants to send a message to its respective Rx.
- The channel is **shared** $\rightarrow$ each Rx sees **interference** from the other Tx.

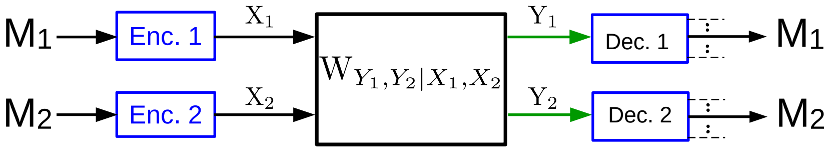
2-User Classical Interference Channel



- Each Tx wants to send a message to its respective Rx.
- The channel is **shared** $\rightarrow$ each Rx sees **interference** from the other Tx.
- **Interference** is similar to **noise**, making decoding harder.

Treating Interference as Noise

Interference is treated as noise; Rx decodes only its own message



- Rxs **ignore interference** and treat it as noise.

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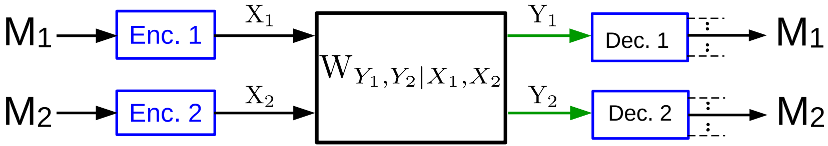

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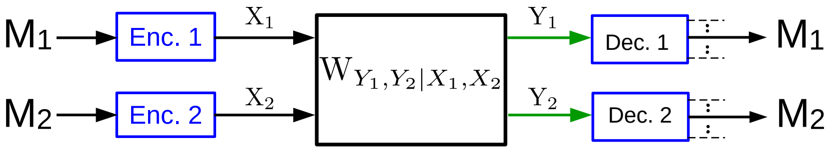
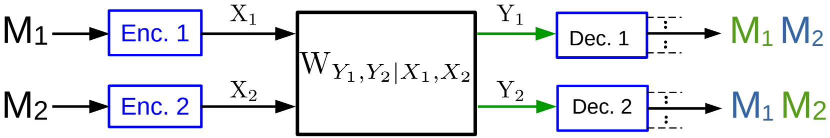


- Rxs **ignore interference** and treat it as noise.
- Each Rx behaves like a point to point decoder
- Simple, but **the achievable rates are small.**

Decoding interference

What if we decode the interference?



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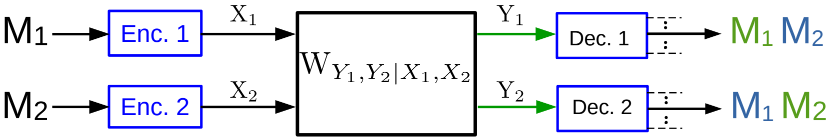

- Each Rx **decodes the interference.**
- **Separate** the desired signal from interference

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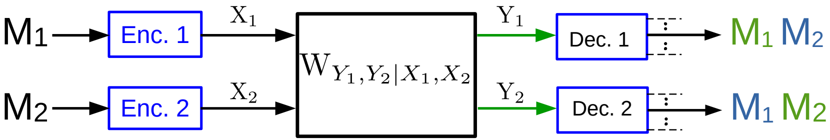
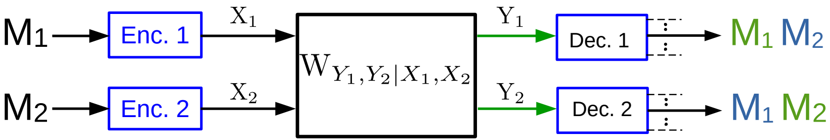


- Each Rx **decodes the interference.**
- **Separate** desired signal from interference
- Achieves higher rates than treating interference as noise.

Decoding interference

What if we decode the interference?

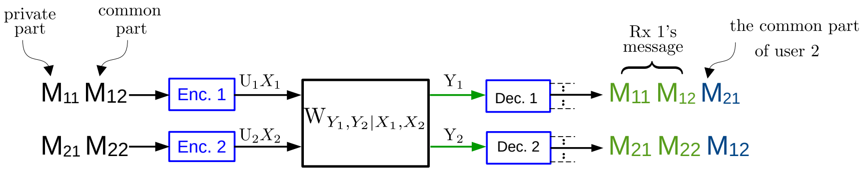


- Rx decodes **more information** than needed.
- Additional decoding constraints reduce achievable rates.

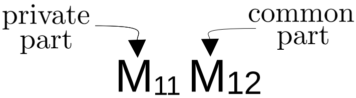
Can we partially decode the interference?

Han Kobayashi coding scheme

Novel scheme: decodes part of interference, ignore the rest



- Each message is split into: **1 private + 1 common part**.



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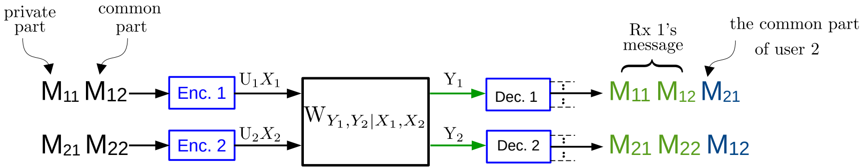

- Each message is split into: **1 private + 1 common part**.
- **Private part:** decoded only by intended Rx.
- **Common part:** decoded by both Rxs.

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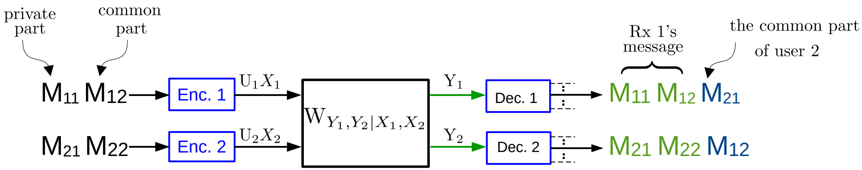
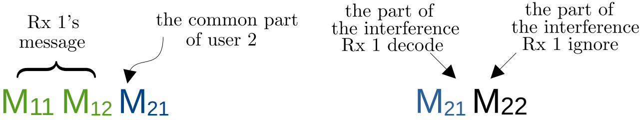


- **Partially decode interference, partially ignore it → improves achievable rates.**



3-User Classical Interference Channel

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- three finite input alphabet sets : $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3$
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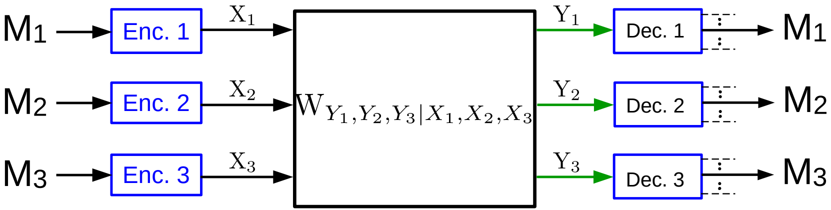
- and a transition probability : $W_{Y_1 Y_2 Y_3|X_1 X_2 X_3}(y_1, y_2, y_3|x_1, x_2, x_3)$



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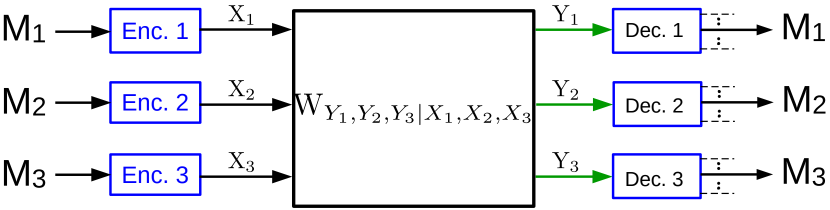
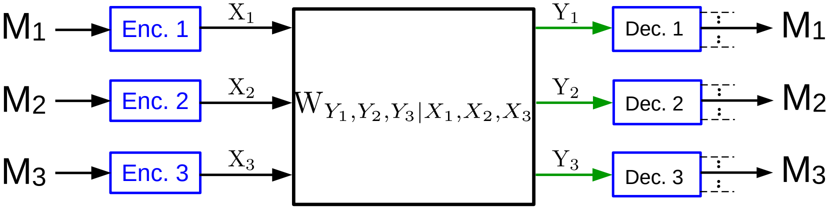


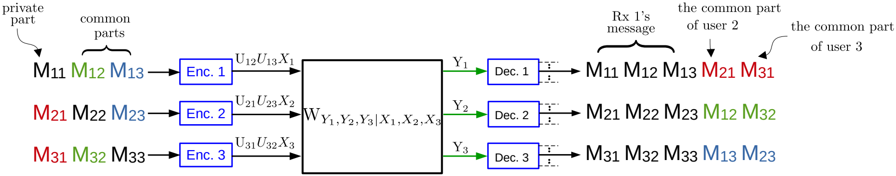
- Three Txs communicate with their respective Rxs over a shared channel.

3-User Classical Interference Channel

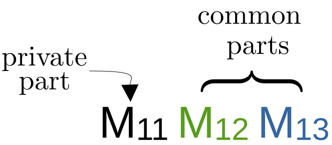


- Three Txs communicate with their respective Rxs over a shared channel.
- Each Rx sees **interference** from the other two **users.**

Han–Kobayashi Scheme for 3-User Interference Channel



- Each message is split into: **1 private part** + **2 common parts.**



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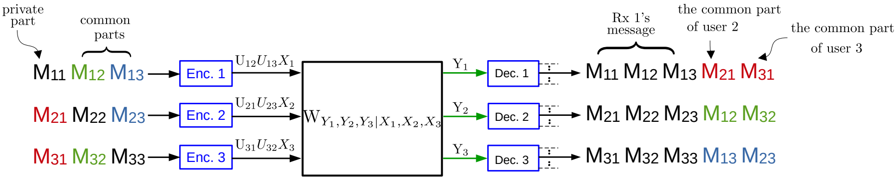

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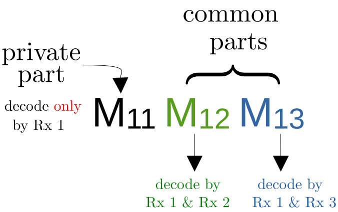

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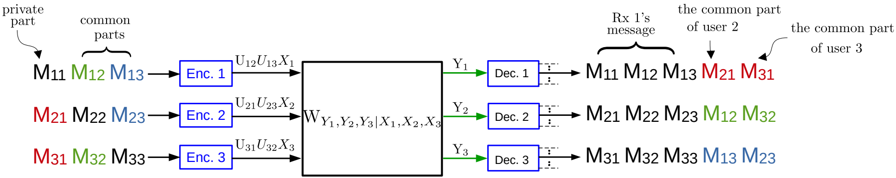
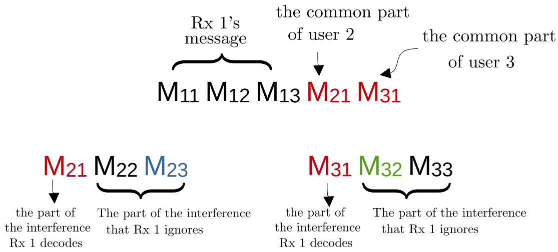


- Each Rx decodes: its own message + the common parts from the two interfering Txs.



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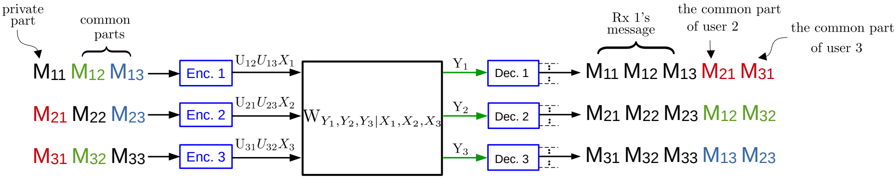
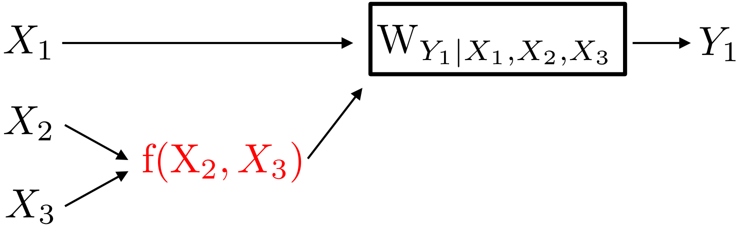


All codes are unstructured i.i.d. codes.

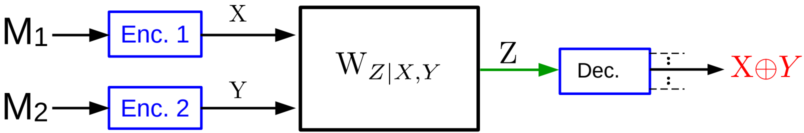
Each Rx decoding only univariate component of the different transmissions.

Interference over 3-User IC is Bivariate



What hurts user 1 is a bivariate function of X_2 and X_3.

Decoding the sum over MAC



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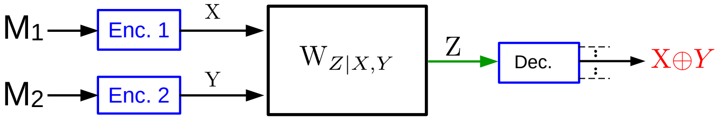
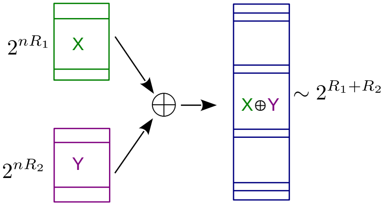
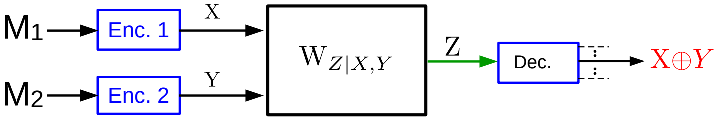


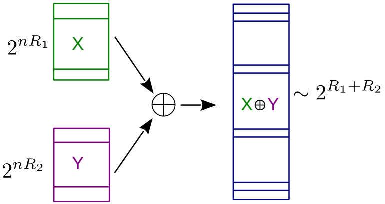


Unstructured i.i.d. random codes explode
in size (range) when added.

i.i.d. codes **only** have empirical properties → hard to decode sums.

Decoding the sum over MAC





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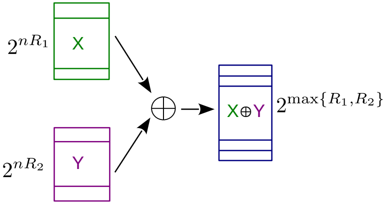
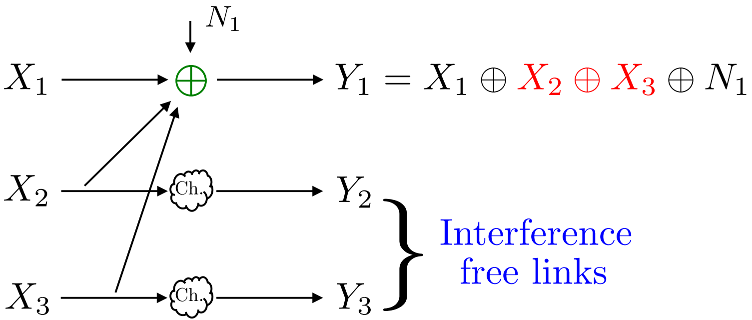


Cosets of the same linear code when added do not explode in size (range).

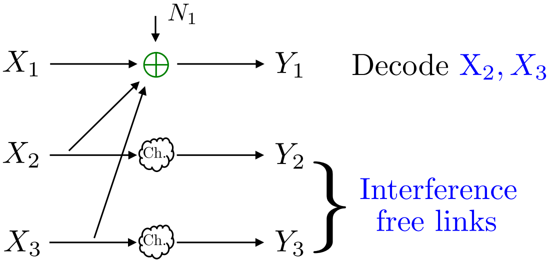
Structured codes have **algebraic properties + empirical properties**.

Role of Coset Codes in 3-user Interference Channel



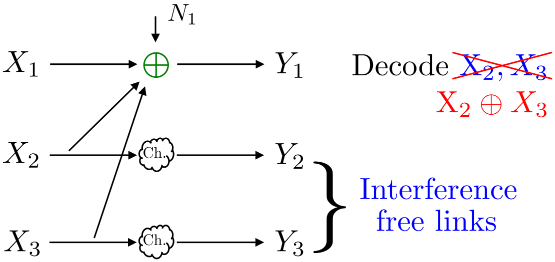
$\bigoplus$ = Logical OR

Role of Coset Codes in 3-user Interference Channel



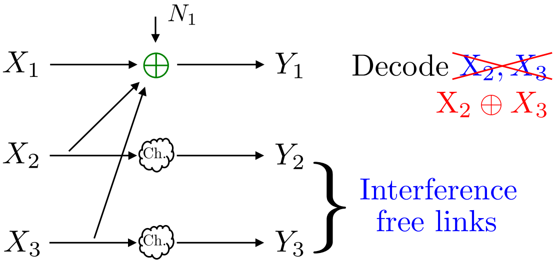
- Using unstructured i.i.d. codes: Decode X_2 and X_3 individually

Role of Coset Codes in 3-user Interference Channel



- Using unstructured i.i.d. codes: Decode X_2 and X_3 individually
- Using coset codes: Directly decode $X_2 \oplus X_3$

Role of Coset Codes in 3-user Interference Channel



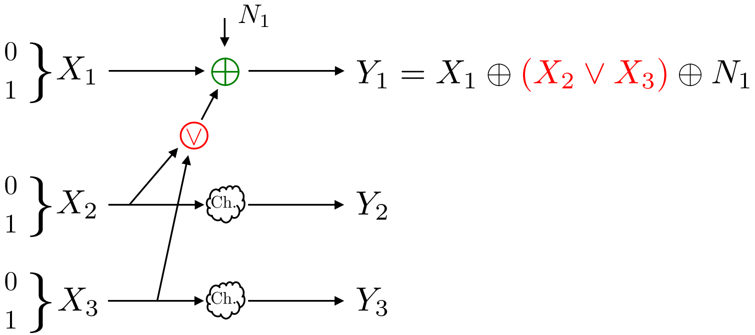
- Using unstructured i.i.d.. codes: Decode X_2 and X_3 individually
- Using coset codes: Directly decode $X_2 \oplus X_3$
- Decoding only the required function can achieve higher rates

Non-Additive Interference

What if the interference is not additive?

Can coset codes still outperform unstructured i.i.d. codes ?

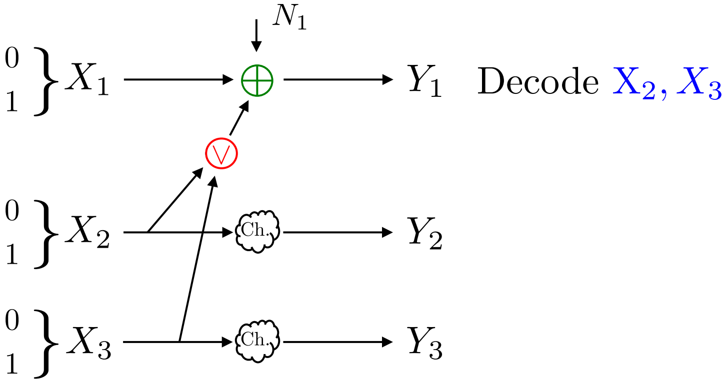
Non-Additive Interference



$\bigoplus$ = Logical OR

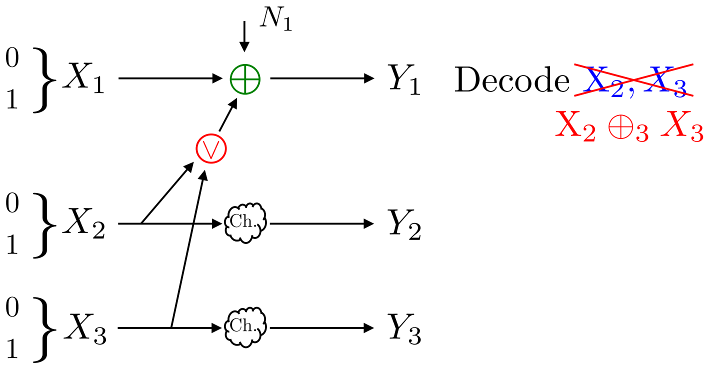
$\bigcirc\!\!\!\!\vee$ = Binary sum, i.e., XOR

Non-Additive Interference



Using unstructured i.i.d. codes: Decode X_2 and X_3 individually

Non-Additive Interference



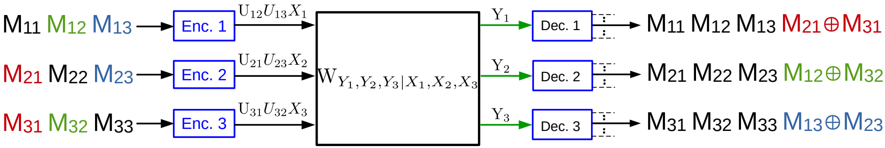
Pretend X_2, X_3 live on ternary field $\{0, 1, 2\}$.

From $X_2 \oplus_3 X_3$, Rx 1 can recover $X_2 \vee X_3$.
Just map 2 to 1 !!

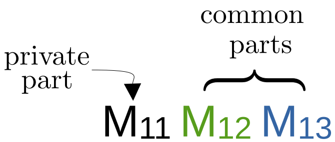
X_2	X_3	$X_2 \oplus_3 X_3$	$X_2 \vee X_3$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	2	1

$\oplus_3$ = Ternary addition.

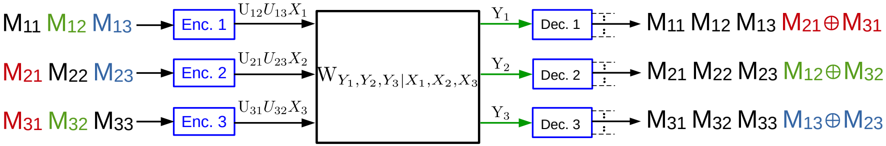
New coding scheme



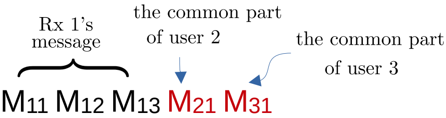
- Each message is split into: **1 private part** + **2 common parts.**



New coding scheme

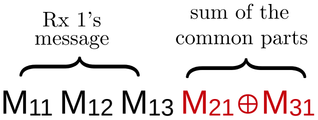


Han Kobayashi coding scheme



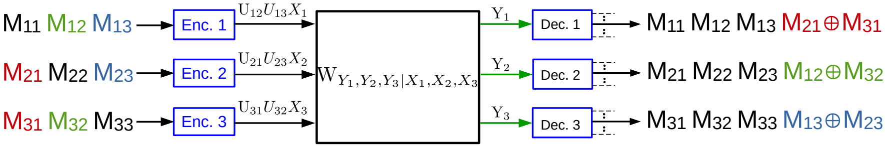
Receiver decodes more information than it actually needs.

Proposed coding scheme



Exploits the joint structure of the interference.

New coding scheme



Han Kobayashi coding scheme

Rx 1's message: M11 M12 M13; the common part of user 2: M21; the common part of user 3: M31

M11 M12 M13 M21 M31

Proposed coding scheme

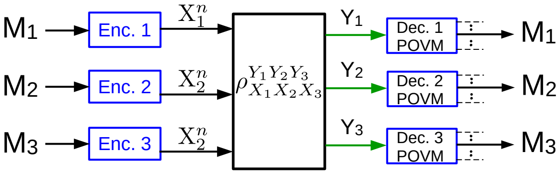
Rx 1's message: M11 M12 M13; sum of the common parts: M21⊕M31

M11 M12 M13 M21⊕M31

Decoding the structured sum leads to higher achievable rates.

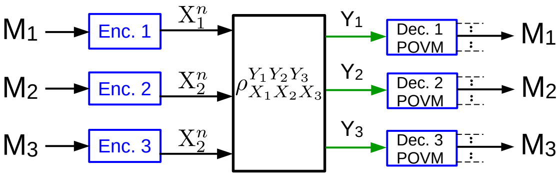
Problem of Interest

3-User Classical Quantum Interference Channel

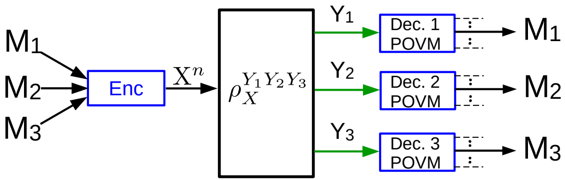


Problem of Interest

3-User Classical Quantum Interference Channel

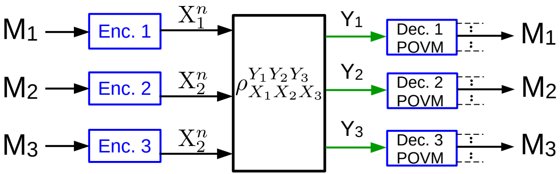


3-User Classical Quantum Broadcast Channel

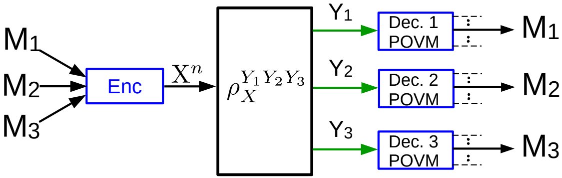


Problem of Interest

3-User Classical Quantum Interference Channel



3-User Classical Quantum Broadcast Channel

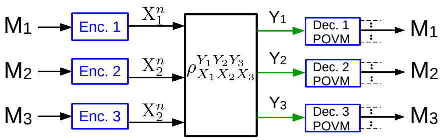


?? Optimal Coding Scheme ?? ?? Capacity Region ??

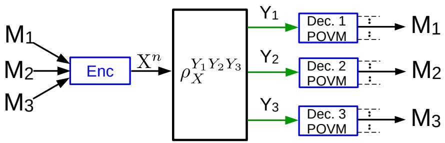
Contributions

All current known coding schemes for

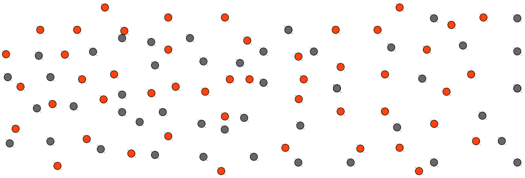
$3-$user CQ Interference Channel $(3-\text{CQIC})$



$3-$user CQ Broadcast Channel $(3-\text{CQBC})$



based on i.i.d. Unstructured codes.

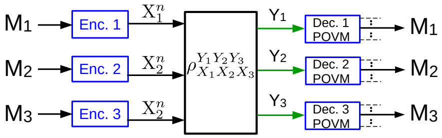


User 1's code. User 2's code.

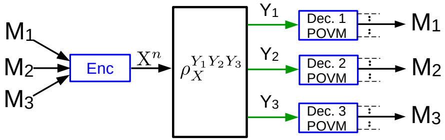
Contributions

New coding schemes for generic

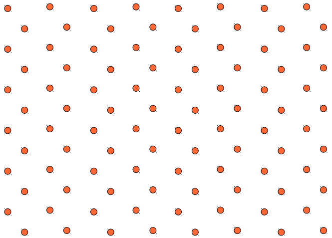
$3-$user CQ Interference Channel $(3-\text{CQIC})$



$3-$user CQ Broadcast Channel $(3-\text{CQBC})$



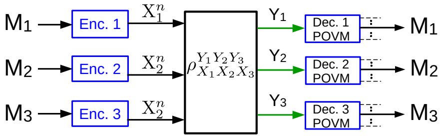
based on COSET codes.



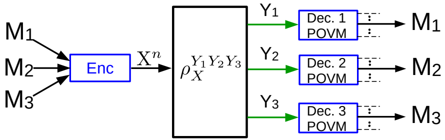
that strictly outperform the best previous known.

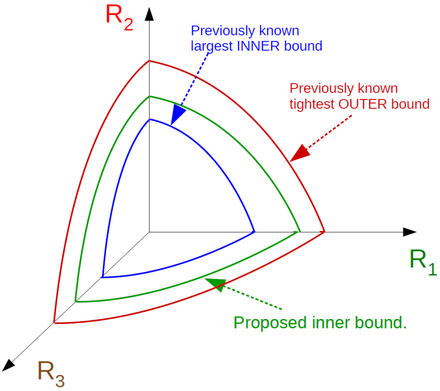
Contributions

$3-$user CQ Interference Channel $(3-\text{CQIC})$



$3-$user CQ Broadcast Channel $(3-\text{CQBC})$



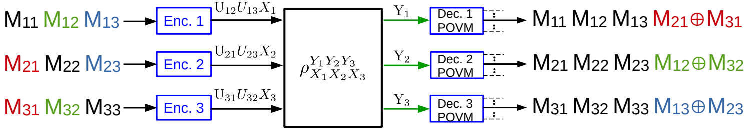


All Known Unstructured IID, i.e. Han-Kobayashi + Marton

Characterized New Inner Bound. Strictly Larger for identified $3-$CQBCs and $3-$CQICs.

The main challenge and contribution

General 3CQIC entails receiver decoding into multiple codebooks.

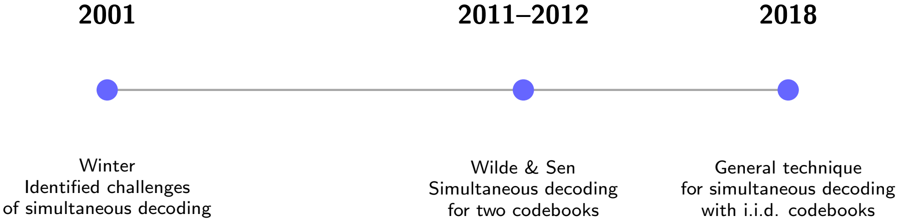


Two approaches for decoding multiple codebooks:

- **Successive Decoding:** Decodes messages one at a time — simpler but suboptimal.
- **Simultaneous Decoding:** Decodes all messages jointly — achieves larger rate regions.

The main challenge and contribution

- Simultaneous decoding is **challenging in CQ channels**. It remained an open problem for nearly two decades before being formally resolved.



The main challenge and contribution

- Simultaneous decoding is **challenging in CQ channels**. It remained an open problem for nearly two decades before being formally resolved.
- Sen's method elegant but **limited to i.i.d. codebooks.**
- Coset codes require new analysis.

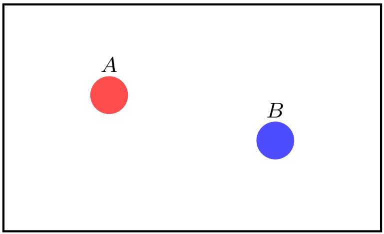
Main contribution: Extend Sen's method to coset codes.

Why simultaneous decoding is challenging in CQ settings

- In the classical setting, simultaneous decoding relies on unions and intersections of typical sets.
- In quantum settings, **typical sets are replaced by subspaces**, and this intuition breaks down.

Why simultaneous decoding is challenging in CQ settings

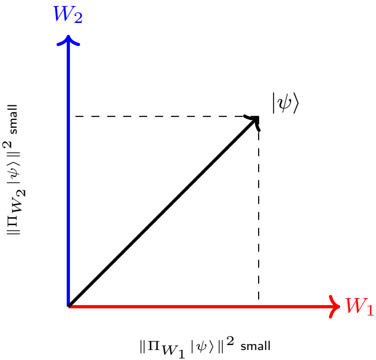
Classical settings



$p(A), p(B) \ll 1$

$p(A \cup B)$ is still small

Quantum settings



$\left\|\Pi_{W_1+W_2} |\psi\rangle\right\|$ large

Thank You for Your Attention

Any questions ?

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