von Mises Belief Propagation

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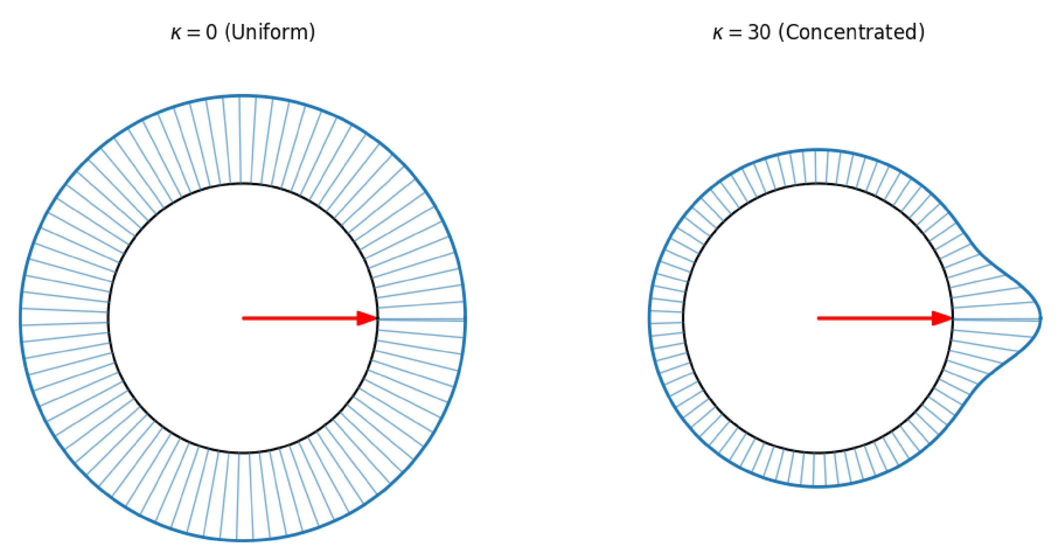
von Mises (vM) Distribution

Let $X \in \mathcal{C} = \{x \in C \text{ s.t. } |x| = 1\}$ be a random variable following vM distribution, the p.d.f. is given by [1]

$$\text{vM}(x;\eta) = \frac{1}{2\pi I_0(\kappa)} \exp\left(\text{Re}(\eta^* x)\right), \quad (1)$$

where $\mu \in \mathcal{C}$ is the mean direction, $\kappa \in [0, \inf)$, $\eta = \kappa\mu$, and $\mu = \eta/|\eta|$ and $\kappa = |\eta|$. The first moment is given by:

$$m_1(\eta) = \int_{x\in\mathcal{C}} x \text{ vM}(x;\eta)\text{d}x = \frac{I_1(\kappa)}{I_0(\kappa)} \frac{\eta}{|\eta|}. \quad (2)$$

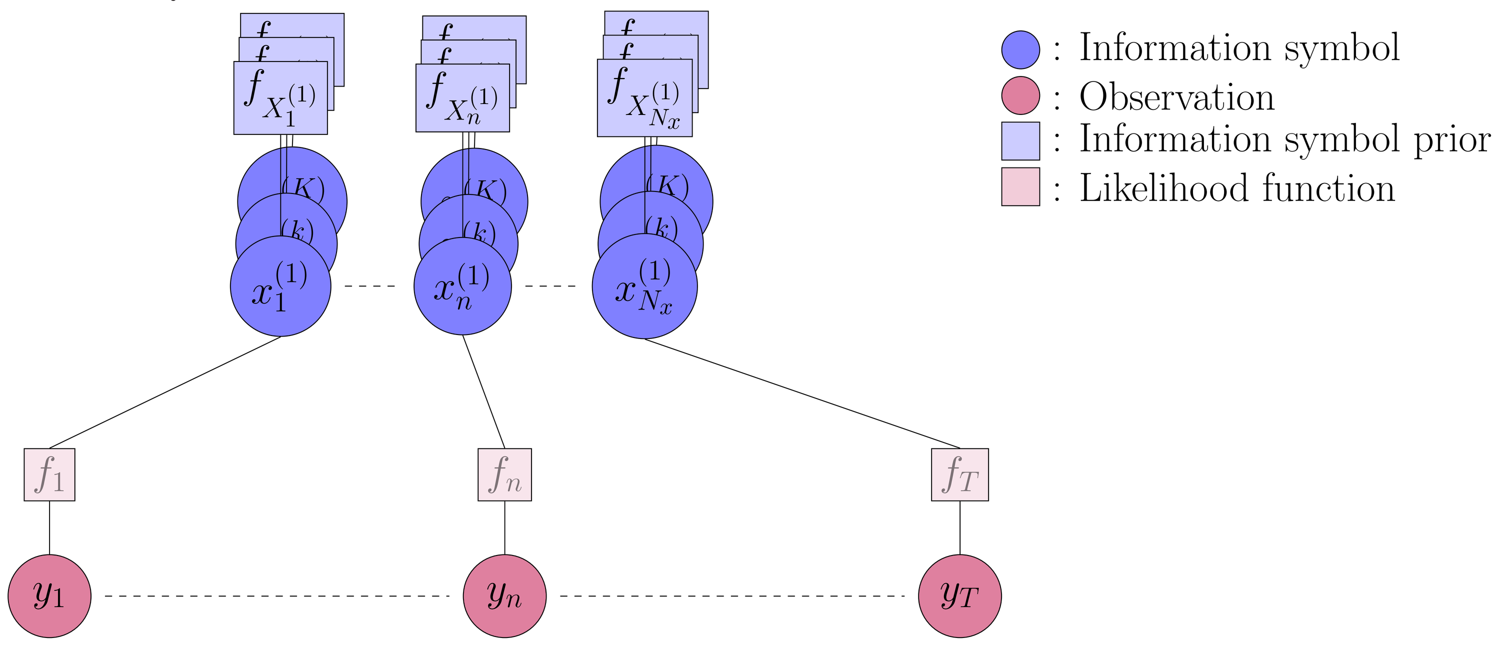


vM-BP for MIMO Detection

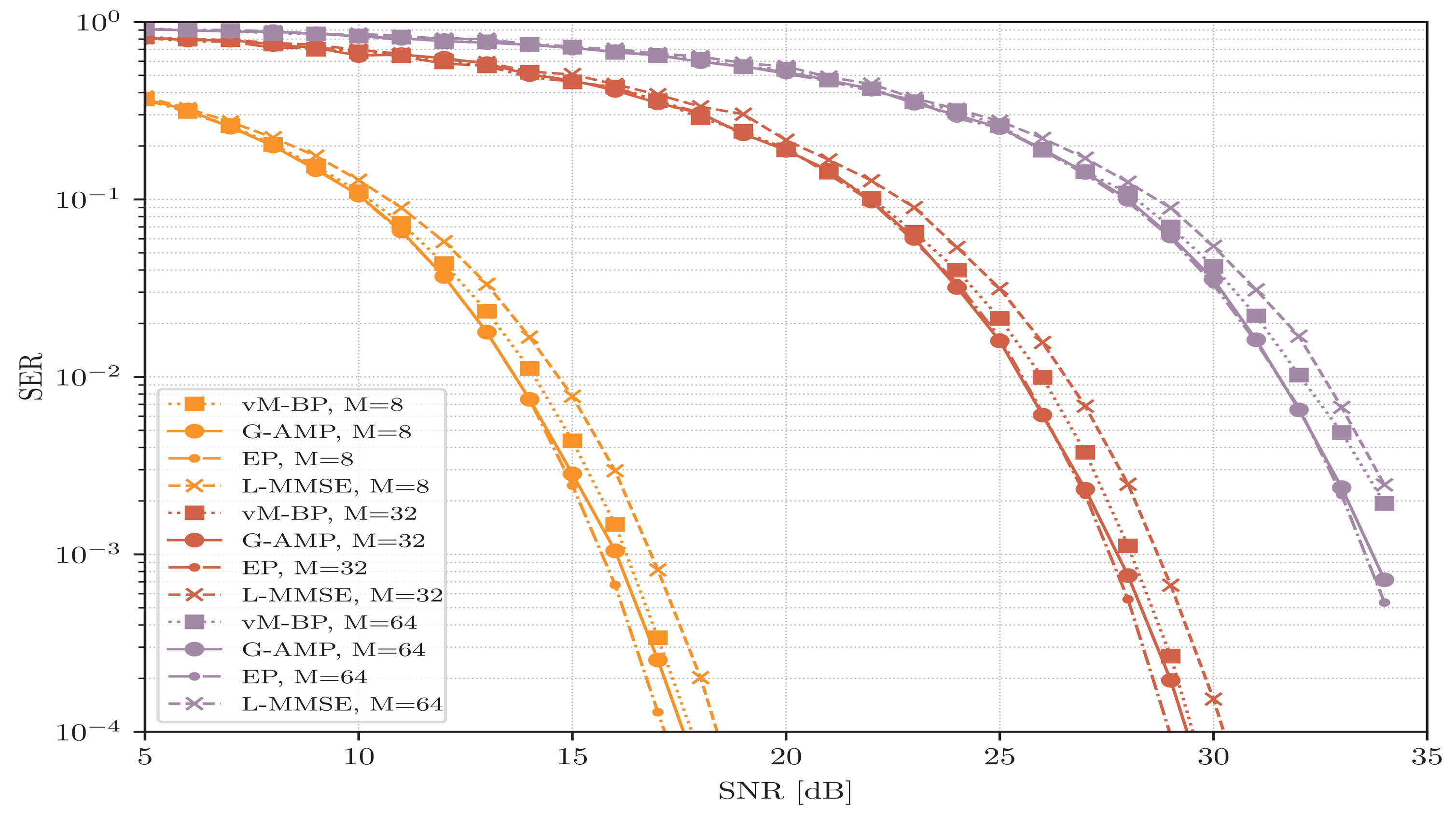
Consider a system with K single-antenna users transmitting T PSK symbols to an N antenna receiver. The optimal MAP detector is exponentially complex in constellation size and the number of symbols transmitted, while low-complexity approaches rely on Gaussian approximations, such as L-MMSE, AMP. We propose a belief propagation (BP) framework exploiting the geometry of the PSK constellation on the unit circle using vM distributions. Let the received signal be

$$\mathbf{Y} = \sum_{k=1}^{K} \mathbf{x}^{(k)}\mathbf{H} + \mathbf{w}, \in C^{N\times T}, \quad (3)$$

where $\mathbf{H}$ is Rayleigh faded i.i.d. channel, $\mathbf{w}$ is AWGN noise and $\mathbf{x}^{(k)} \in \mathcal{C}^T$. The factor graph consisting of both variable nodes (VN) and function nodes (FN), for this system model is given below and the types of FN present are: Gaussian prior of channel nodes: $f_{H_n^{(k)}}$; Uniform vM prior of information symbols: $f_{X_i^{(k)}}$; and Likelihood according to (3): f_t.



Symbol error rate (SER) results for 16 users, 64 receive antennas for different modulation indices M are present below with comparison to several existing iterative algorithms.



von Mises Belief Propagation (vM-BP)

1. Types of Messages:
In any given factor graph, at a given iteration itr, there are two types of messages:

- VN (v) to FN (f): $\lambda_{v\to f}^{[itr]} = \prod_{f'\in\mathcal{N}(v)\sim f} \lambda_{f'\to v}^{[itr-1]}$.
- FN (f) to VN (v): $\lambda_{f\to v}^{[itr]} = \int_{\mathcal{N}(f)\sim v} f \prod_{v'\in\mathcal{N}(f)\sim v} \left(\lambda_{v'\to f}^{[itr-1]} dv'\right)$

2. Proposed Approximations:
The messages associated with PSK symbols are approximated using vM distributions and the channel messages are approximated using complex Gaussian distributions. The approximation and parameterization result in closed form simple messages while the general discrete messages have more complexity. The message from VN $x_i^{(k)}$ (or $h_n^{(k)}$) to FN f_t at iteration itr and the message from FN f_t to VN $x_i^{(k)}$ (or $h_n^{(k)}$) approximated using [2] are presented below.

$$\lambda_{x_i^{(k)}\to f_t}^{[itr]}(\,\cdot\,) \propto \text{vM}(\,\cdot\,;\eta_{x_i^{(k)}\to f_t}^{[itr]}). \quad (4)$$

$$\lambda_{h_n^{(k)}\to f_t}^{[itr]}(\,\cdot\,) \propto \text{CN}(\,\cdot\,;\mu_{h_n^{(k)}\to f_t}^{[itr]}, \sigma_{h_n^{(k)}\to f_t}^{2[itr]}). \quad (5)$$

$$\lambda_{f_t\to x_i^{(k)}}^{[itr]}(\,\cdot\,) \approx \exp\left(E_{\mathcal{N}(f_t)\sim x_i^{(k)}}[\log(f_t(\mathbf{x}_i, \mathbf{h}_n; y_t))]\right) = vM(\,\cdot\,;\eta_{f_n\to x_i^{(k)}}^{[t]}). \quad (6)$$

$$\lambda_{f_t\to h_n^{(k)}}^{[itr]}(\,\cdot\,) \approx CN(\,\cdot\,;\mu_{f_t\to h_n^{(k)}}^{[itr]}, \sigma_{f_t\to h_n^{(k)}}^{2[itr]}). \quad (7)$$

The beliefs of VN v (be it information symbol or channel node) at iteration itr is written as

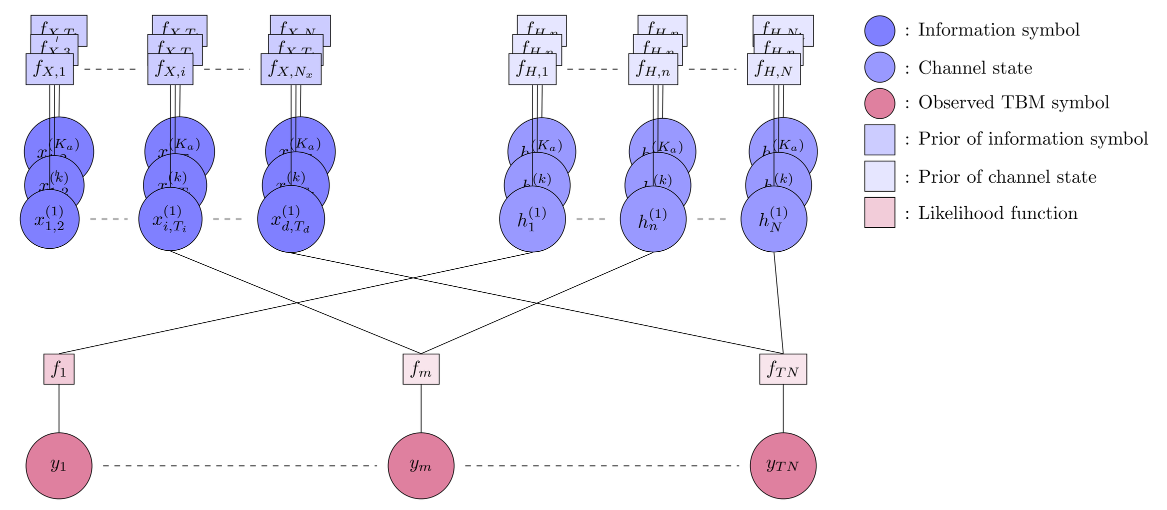
$$b_v^{[itr]}(\,\cdot\,) \propto f_V(v) \prod_{t'\in\mathcal{N}(v)} \lambda_{f_{t'}\to v}^{[itr]}(\,\cdot\,). \quad (8)$$

vM-BP for unsourced random access

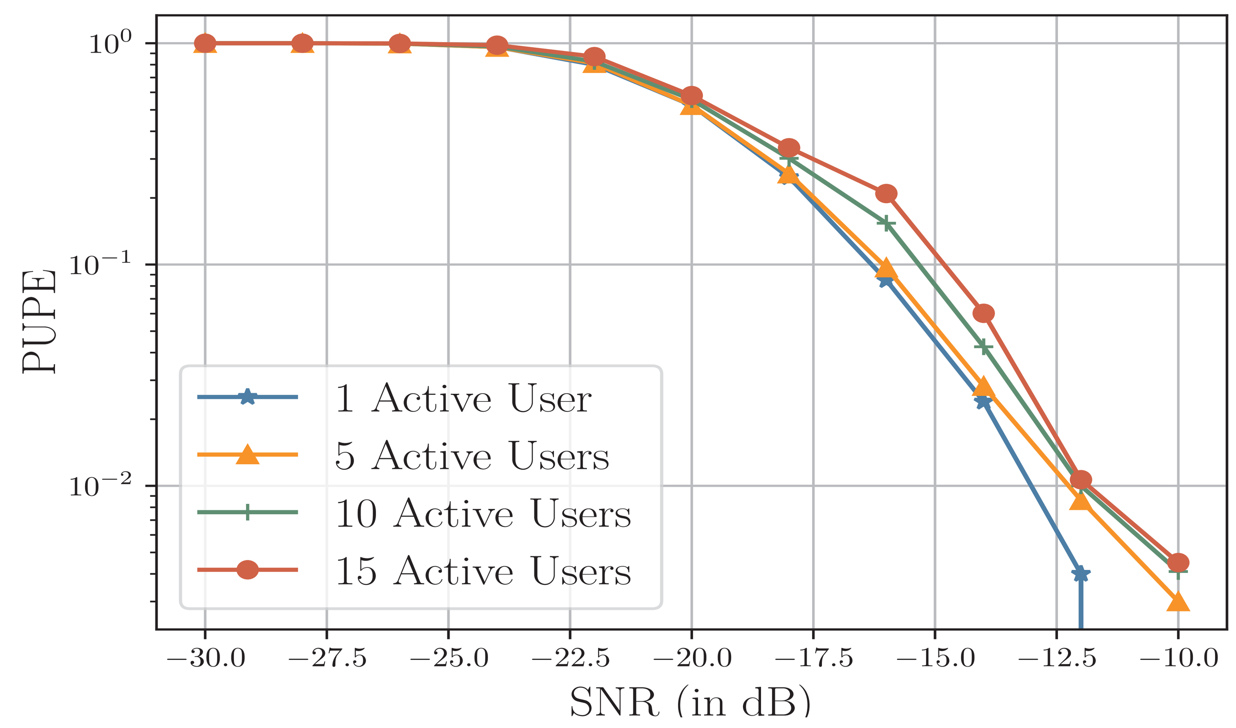
The algorithm vM-BP can be applied to any factor graph where the symbols are PSK modulated. One such application is unsourced random access (URA) using tensor-based modulation (TBM) [3], where the received signal for TBM is given as

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{x}_1^{(k)} \otimes \mathbf{x}_2^{(k)} \otimes \ldots \otimes \mathbf{x}_d^{(k)} \otimes \mathbf{h}^{(k)} + \mathbf{w}, \quad (9)$$

where $\mathbf{x}_1^{(k)} \otimes \mathbf{x}_2^{(k)} \otimes \ldots \otimes \mathbf{x}_d^{(k)} \in C^{\prod_i T_i}$ is the tensor transmitted from user k, and $\mathbf{h}_k \in C^N$ is the channel vector of each user. The factor graph for such a system is given below.



The per user probability of error (PUPE) results for a system with different active users, 5 receive antennas and 3200 channel uses are given below.



References

[1] K. V. Mardia and P. E. Jupp, *Directional statistics* (Wiley Series in Probability and Statistics). John Wiley & Sons, 2009.

[2] J. Winn and C. M. Bishop, "Variational message passing," *Journal of Machine Learning Research*, vol. 6, no. 23, pp. 661–694, 2005.

[3] A. Decurninge, I. Land, and M. Guillaud, "Tensor-based modulation for unsourced massive random access," *IEEE Wireless Commun. Letters*, vol. 10, no. 3, pp. 552–556, 2021.